Computation of Extended Suzuki Mobile Fading Channel (Type I) Parameters

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ABSTRACT

The analysis and modeling of mobile fading communications channels has been an active research topic since early beginnings of mobile radio systems. In mobile wireless communication systems, the multipath propagation in addition to the movement of the receiver and/or the transmitter leads to drastic and random fluctuations of the received signal. For flat fading channels, the extended Suzuki process of type (I) has been proposed as a stochastic model of such received signal fluctuations. In this paper we discuss this stochastic model and an appropriate deterministic model, and successively present and analyze simulation results based on these models.

Keywords: Wireless, Communications, Fading, Channels.

1. INTRODUCTION

The electromagnetic wave propagation in mobile communication systems is greatly affected by the terrain and surroundings. As a result, the propagated signals will possibly meet with multifold reflection, diffraction and scattering which make the propagated signals multipath fading. Besides the multipath propagation, also the Doppler Effect has a negative influence on the transmission characteristics of the mobile radio channel. Due to the movement of the mobile unit, the Doppler Effect causes a frequency shift of each of the partial waves. Hence, the spectrum of the transmitted signal undergoes a frequency expansion during transmission. This effect is called frequency dispersion. Suzuki processes can be considered to be models for the random fluctuations of the received signal in flat-fading land mobile radio systems. The Suzuki processes are obtained by combining stationary normal random processes assumed to be statistically independent. Often this assumption does not meet the real conditions in multipath wave propagation. Therefore, modified and extended Suzuki processes with cross-correlation and dependence between the normal processes have been introduced [1],[2],[3],[5].

The Extended Suzuki process of Type I is one of the Channel model processes to be applied to flat-fading (that is frequency nonselective fading) channels. This process results from the product of Rice process $\xi(t)$ and a lognormal process $\lambda(t)$ i.e.

$$\eta(t) = \xi(t) \cdot \lambda(t)$$

(1)

The long-term fading signal is here modeled by lognormal process $\lambda(t)$ taking the slow time variation of the average local received power into account, whereas the Rice process $\xi(t)$ models the short-term fading of the received signal. The extended Suzuki process $Hu(t)$ is suitable as a stochastic model for a large class of satellite and land mobile radio channels in environments, where a direct line-of sight connection between the transmitter and the receiver cannot be ignored. The probability density function $p_\eta(t)$ of the extended Suzuki process $\eta(t)$ can be calculated by means of the relation

$$p_\eta(z) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi} \sigma_\psi} \rho(z, y) dy$$

(2)

where $p_{\lambda,\psi}(\frac{z}{\sigma_\psi}, y)$ is the joint probability density function of the processes $\xi(t)$ and $\lambda(t)$ at the same instant $t$. Assuming that these two processes are independent, we can find the following integral equation for the probability density function of extended Suzuki processes [2]

$$p_\eta(z) = \frac{1}{\sqrt{2\pi} \sigma_\psi^2} \int_{0}^{\infty} \frac{(\frac{z}{\sqrt{2}\sigma_\psi})^2 + \sigma_\psi^2}{2\sigma_\psi^2} e^{-\frac{(\psi_\eta(z) - \sigma_3)^2}{2\sigma_3^2}}$$

(3)

where $\sigma_3 > 0$, and $\psi_\eta$ is the average power of the two real-valued Gaussian random processes determining the Rice process $\xi(t)$. The parameter $\sigma$ represents the amplitude of the line of sight component $m(t)$, while $\sigma_3$ and $m_3$ are the statistical parameters of the long term fading as will be discussed in section 3 of this paper. From Eq. 3, it can be seen that $p_\eta(z)$ depends on $\psi_\eta$, $\rho$, $\sigma_3$, and $m_3$.

2. MODELLING OF SHORT TERM FADING

The stochastic reference model for extended Suzuki processes of Type I is shown in Fig. 1, where the short-term fading is modeled by the Rice process given by

$$\xi(t) = |\mu \rho(t)| = |m(t) + \mu(t)|$$

(4)
where,
\[ \mu(t) = \mu_1(t) + j\mu_2(t) \]  
represents the sum of all scattered non-line-of-sight components of the received signal over the mobile fading channel, and
\[ m(t) = m_1(t) + jm_2(t) = \rho e^{j(2\pi f_d t + \theta)} \]  
is the line-of-sight signal component, where \( \rho, f_d, \) and \( \theta \) denote the amplitude, the Doppler frequency, and the phase of the line-of-sight component, respectively.

We assume that by spatially limited obstacles or by using directional antennas or sector antennas, (i.e., antennas with non-omnidirectional radiation patterns), no electromagnetic waves with angles or arrival within the interval from \( \alpha_0 \) to \( 2\pi - \alpha_0 \) can arrive at the receiver, where \( 2\pi/2 \leq \alpha_0 \leq 3\pi/2 \)

![Fig.1: Reference Stochastic model of extended Suzuki processes - Type 1.](image)

The resulting unsymmetrical Doppler power spectral density \( S_{\mu\mu}(f) \) is then described as follows
\[ S_{\mu\mu} = \begin{cases} 0 & \text{else} \\ \frac{\sigma_0^2}{f_{\text{max}}^2} & -f_{\text{min}} \leq f \leq f_{\text{max}} \\ \frac{\sigma_0^2}{f_{\text{min}}^2} (1 - \left(\frac{f}{f_{\text{max}}}\right)^2) & \text{else} \end{cases} \]  

where \( f_{\text{max}} \) denotes the maximum Doppler frequency, and \( f_{\text{min}} = f_{\text{max}} \cos(\Theta) \) which lies in the range \( 0 \leq f \leq f_{\text{max}} \). In general, the shape of \( S_{\mu\mu} \) is unsymmetrical, which results in a non-zero cross-correlation of the real-valued Gaussian random processes \( \mu_1(t) \) and \( \mu_2(t) \). From Fig.1, the following relations hold for the underlying processes:
\[ \mu_1(t) = \nu_1(t) + \nu_2(t) \]  
\[ \mu_2(t) = \nu_1(t) + \nu_2(t) \]  
where \( \mu_i(t) \) represents a colored Gaussian random process, and \( \nu_i(t) \) is the Hilbert transform of \( \nu_i(t) \) for \( i = 1, 2 \). Here the spectral shaping of \( \nu_i(t) \) is based on filtering of white Gaussian noise by using an ideal filter whose transfer function is given by
\[ H_i(f) = \sqrt{\frac{\sigma_0^2}{\tau} S_i(f)} \]  
The autocorrelation function of \( \mu(t) = \mu_1(t) + j\mu_2(t) \) can be expressed in terms of the autocorrelation and cross-correlation of \( \mu_1(t) \) and \( \mu_2(t) \) as follows:
\[ R_{\mu\mu}(\tau) = R_{\mu_1\mu_1}(\tau) + jR_{\mu_1\mu_2}(\tau) + j\mu_2(t) \]
\[ + R_{\mu_2\mu_1}(\tau) + j\mu_1(t) \]
which can be expressed as
\[ R_{\mu\mu}(\tau) = 2[R_{\nu_1\nu_1}(\tau) + R_{\nu_2\nu_2}(\tau) + j(R_{\nu_1\nu_2}(\tau) - R_{\nu_2\nu_1}(\tau))] \]  

After Fourier transforming \( R_{\mu\mu}(\tau) \), the Doppler power spectral density can be given as:
\[ S_{\mu\mu}(f) = 2[|1 - \text{sgn}(f)|S_{\nu_1\nu_1}(f) + (1 - \text{sgn}(f))S_{\nu_2\nu_2}(f)] \]  
which can be expressed as:
\[ S_{\mu\mu}(f) = 2(|1 - \text{sgn}(f))S_{\nu_1\nu_1}(f) + (1 - \text{sgn}(f))S_{\nu_2\nu_2}(f)] \]  

3. MODELLING OF LONG TERM FADING

Measurements have shown that in many wireless communication systems the statistical behavior of long-term fading is quite similar to a lognormal process \( \nu_3(t) \). With such a process, the slow fluctuation of the local mean value \( \lambda(t) \) of the received signal, which is determined by shadowing effects, is given by
\[ \lambda(t) = e^{\sigma_3^2 \nu_3(t) + m_3} \]  
where \( \nu_3(t) \) is a real-valued Gaussian random process with mean of zero and variance of unity. The model parameters \( m_3 \) and \( \sigma_3 \) can be used in connection with the parameters of the Rice process \( (\sigma_0^2, f_{\max}, f_{\min}, \rho, \theta) \) to fit the model behavior to the statistics of real-world channels. We assume that the stochastic process \( \nu_3(t) \) is statistically independent of the processes \( \nu_1(t) \) and \( \nu_2(t) \). \( S_{\nu_3\nu_3}(f) \) is assumed to have the form of the Gaussian power spectral density
\[ S_{\nu_3\nu_3}(f) = \frac{1}{\sqrt{2\pi\sigma_3^2}} e^{-\frac{f^2}{2\sigma_3^2}} \]

The 3-dB-cut-off frequency \( f_c = \sigma_3 \sqrt{2\ln 2} \) is in general much smaller than the maximum Doppler frequency \( f_{\max} \). The autocorrelation function of the process \( \nu_3(t) \) can be expressed as
\[ R_{\nu_3\nu_3}(\tau) = e^{-2(\pi\sigma_3^2\tau)^2} \]
which corresponds to the inverse Fourier transform of $S_{\alpha}(f)$. The autocorrelation function $R_{\alpha\alpha} (\tau)$ of the lognormal process can be expressed in terms of $R_{\alpha\alpha} (\tau)$ as

$$R_{\alpha\alpha} (\tau) = e^{2m_{\alpha} + \sigma_{\alpha}^2 |1 + R_{\alpha\alpha} (\tau)|}$$  \hfill (17)

The power spectral density $S_{\alpha\alpha} (f)$ can now be expressed in terms of the power spectral density $S_{\alpha\alpha} (f)$ as follows

$$S_{\alpha\alpha} (f) = e^{2m_{\alpha} + \sigma_{\alpha}^2 |\delta(f) + \sum_{n=1}^{\infty} \frac{\sigma_{\alpha}^2}{n!} \frac{S_{\alpha\alpha} (f) / \sqrt{n}}{\sqrt{n}}|}$$  \hfill (18)

where $\delta(f)$ is the Dirac-Delta function.

4. IV. DETERMINISTIC SIMULATION MODEL FOR THE EXTENDED SUZUKI PROCESS OF TYPE I

Stochastic multipath propagation models for indoor and out-door mobile radio channels are in general derived by employing colored Gaussian noise processes. Efficient design and realization techniques of such processes are therefore of particular importance in the area of mobile radio channel modeling. Figure 2 shows the deterministic simulation model for extended Suzuki process of Type I that approximates the behavior of the stochastic reference model shown in Fig. 1. From the previous sections, it was seen that the reference model for the extended Suzuki process of Type I is based on the use of three real-valued Gaussian random processes $\nu_i(t)$ or $\mu_i(t), (i=1,2,3)$.

The ideal Gaussian random process $\nu_i(t)$ can be approximated deterministically by

$$\hat{\nu}_i(t) = \sum_{n=1}^{N_i} c_{i,n} \cos(2\pi f_{1,n} t + \theta_{i,n}), \quad i = 1, 2, 3$$  \hfill (19)

where $N_i$ is a sufficiently large integer (ideally $\infty$). The corresponding autocorrelation function is:

$$R_{\nu_i\nu_i} (\tau) = \sum_{n=1}^{N_i} \frac{c_{i,n}^2}{2} \cos(2\pi f_{1,n} \tau), \quad i = 1, 2, 3$$  \hfill (20)

The corresponding power spectral density of $\nu_i(t)$ with $i=1,2,3$ is given as

$$S_{\nu_i\nu_i} (f) = \sum_{n=1}^{N_i} \frac{c_{i,n}^2}{4} |\delta(f - f_{1,n}) + \delta(f - f_{i,n})|$$  \hfill (21)

Using the mean-square-error method (MSEM)[2] to calculate the Doppler coefficients $c_{i,n}$ and $f_{i,n}$ for $i = 1, 2$ we get

$$c_{i,n} = 2\sigma_0 \sqrt{\frac{1}{T_{\text{max}}} \int_{0}^{T_{\text{max}}} j_0(2\pi f_{\text{max}} \tau) \cos(2\pi f_{i,n} \tau) d\tau}$$  \hfill (22)

where $T_{\text{max}} = 1/(2f_{\text{max}})$. There is no closed-form solution for the definite integral in this equation, so numerical integration has to be applied to calculate the coefficients $c_{i,n}$. The Doppler frequencies are

$$f_{i,n} = \left\{ \begin{array}{ll}
\frac{f_{\text{max}}}{N_1}, & i = 1, \quad n = 1, 2, \ldots, N_1 \\
\frac{f_{\text{max}}}{N_2}, & i = 1, \quad n = 1, 2, \ldots, N_2
\end{array} \right.$$  \hfill (23)

where

$$N_2 = \frac{N_2}{\pi \arcsin(\kappa_0)}$$  \hfill (24)

is an auxiliary variable that depends on the frequency ratio $\kappa_0 = f_{\text{min}} / f_{\text{max}}$. The actual required number of harmonic functions $N_2 (\leq N_2)$ which is necessary for the realization of $\hat{\nu}_2(t)$ is defined by the user.

The computation of the discrete Doppler coefficients $c_{3,n}$ of the third Gaussian process $\hat{\nu}_3(t)$, whose power spectral density is Gaussian shaped, are given for $n=1,2,\ldots,N_3$ as

$$c_{3,n} = 2\sigma_0 \sqrt{\frac{1}{T_{\text{max}}} \int_{0}^{T_{\text{max}}} e^{-\frac{(\pi f_{\text{max}} \tau)}{2}} \cos(2\pi f_{3,n} \tau) d\tau}$$  \hfill (25)

and

$$f_{3,n} = \frac{\kappa_c f_c}{2N_3 (2n - 1)}$$  \hfill (26)

where the quantity $\kappa_c$ is chosen in such that the mean power of Gaussian power spectral density obtained within the frequency range $|f| \leq \kappa_c f_c$ makes up at least 99.90% of its total mean power. This requirement can be achieved with $\kappa_c = 2 \sqrt{2/\ln 2}$. 

\textbf{Fig. 2: Simulation model for the deterministic extended Suzuki processes of Type I.}
5. SIMULATION RESULTS

Figure 3 shows the power spectral density (as determined by the mean-square-error method) and the autocorrelation function of $\nu_1(t)$, where the number of harmonic functions $N_1$ is considered as 17 (ideally $N_1 = \infty$), and the maximum Doppler frequency $f_{\text{max}}$ is 85Hz [5]. The difference between the autocorrelation function of the reference model and the simulation model decreases by increasing the number of harmonic functions ($N_1$).

A similar distribution shape for the power spectral density of $\nu_2(t)$ can be seen in Figure 4, where the number of harmonic functions ($N_2$) is calculated according to Eq. (24) [5]. The power spectral density of $\nu_3(t)$ shown in Fig. 5, represents the Gaussian power spectral density given by eq. 15. Here again the difference between the autocorrelation function of the reference model and the simulation model decreases by increasing the number of harmonic functions $N_3$.

![Figure 3: Power spectral density $S_{11}(f)$ and Autocorrelation function $R_{11}(\tau)$ with $N_1=17$, $f_{\text{max}}=85$Hz, $\sigma^2=1$.](image)

![Figure 4: Power spectral density $S_{22}(f)$ and Autocorrelation function $R_{22}(\tau)$ with $N_2=17$, $f_{\text{max}}=85$Hz, $\sigma^2=1$.](image)

![Figure 5: Power spectral density $S_{33}(f)$ and Autocorrelation function $R_{33}(\tau)$ with $N_3=17$, $f_{\text{max}}=85$Hz, $f_c = \sqrt{\ln 2} f_{\text{max}}$.](image)

The parameters $\sigma_0$, $\kappa_0$, $\rho_0$, $\sigma_3$, $m_3$, $f_0/f_{\text{max}}$ were experimentally optimized in [2] for heavy and light shadowing, the optimized values are shown in Table 1. Using these values of the parameters and assumed values for $N_1$, $N_2$, $N_3$, and $f_{\text{max}}$ in simulating the extended Suzuki process of type I, we get Figure 6 for heavy shadowing regions and Figure 7 for light shadowing regions [5], [7].

From these two figures, it can be seen that the average signal level for heavily shadowed line-of-sight component is smaller than that for lightly shadowed line-of-sight component.

Also, the deep fades for heavy shadowing regions are much larger than that for light shadowing regions. These results are expected because as the strength of the line-of-sight component increases, it dominates the received signal; hence the effect of fading decreases.

![Figure 6: Simulation of the extended Suzuki process $\eta(t)$ of Type I for heavy shadowing regions, $N_1=17$, $N_2=0, N_3=17$, $f_{\text{max}}=85$Hz.](image)
Table 1: The optimized parameters of the reference channel model for areas with heavy and light shadowing.

<table>
<thead>
<tr>
<th>Shadowing</th>
<th>( \sigma_0 )</th>
<th>( \kappa_0 )</th>
<th>( \rho_0 )</th>
<th>( \sigma_3 )</th>
<th>( m_3 )</th>
<th>( f_\rho / f_{max} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heavy</td>
<td>0.2022</td>
<td>2.40E-11</td>
<td>0.1118</td>
<td>0.1175</td>
<td>0.4906</td>
<td>0.6306</td>
</tr>
<tr>
<td>High</td>
<td>0.4407</td>
<td>5.90E-08</td>
<td>0.0856</td>
<td>0.0101</td>
<td>0.0875</td>
<td>0.7326</td>
</tr>
</tbody>
</table>

Fig.7: Simulation of the extended Suzuki process \( \eta(t) \) of Type I for light shadowing, \( N_1=17, N_2=0, N_3=17, f_{max} = 85Hz \).

6. EXTENDED SUZUKI MODEL (TYPE I) VERSUS EXTENDED SUZUKI MODEL (TYPE II)

The stochastic reference model for extended Suzuki processes of Type II is shown in Fig. 8, where the short-term fading is modeled by the process

\[ \zeta(t) = |\mu(t)| = |m(t) + \mu(t)| \]  

(27)

where,

\[ \mu(t) = \mu_1(t) + j\mu_2(t) \]  

(28)

represents the sum of all scattered non-line-of-sight components of the received un-modulated carrier signal over the mobile fading channel, and

\[ m(t) \quad m_1(t) \quad j m_2(t) = \rho e^{j(2\pi f_\rho t + \theta_\rho)} \]  

(29)

is the line-of-sight signal component, where \( \rho, f_\rho, \) and \( \theta_\rho \) denote the amplitude, the Doppler frequency, and the phase of the line-of-sight component, respectively.

In order to simplify the model, we will assume that the Doppler frequency of the line-of-sight component is equal to zero, hence the line-of-sight component can be expressed as

\[ m(t) \quad m_1 \quad j m_2 = \rho e^{j(\theta_\rho)} \]  

(30)

As a result, the stochastic process \( \xi(t) \) which will be called the extended Rice process, can be given as

\[ \xi(t) = |\mu_\rho| = \sqrt{(m_1(t) + m_1)^2 + (m_2(t) + m_2)^2} \]  

(31)

The Doppler power spectral density \( S_{\nu_\rho}(f) \) of the process \( \nu_\rho(t) \) is described by

\[ S_{\nu_\rho}(f) = \left\{ \begin{array}{ll} \frac{\sigma_\rho^2}{\pi f_{max} \sqrt{1 - (f/f_{max})^2}} & , |f| \leq \kappa_0 f_{max} \\ 0 & , |f| > \kappa_0 f_{max} \end{array} \right. \]  

(32)

where \( f_{max} \) denotes the maximum Doppler frequency, and the variable \( 0 < \kappa_0 \leq 1 \) gives a simple and effective method to reduce the Doppler spread of \( S_{\nu_\rho}(f) \), to make it more close to reality. From Fig.8, the following relations hold for the underlying processes:

\[ \mu_1(t) = \nu_0(t) \]  

(33)

\[ \mu_2(t) = \cos(\theta_0)\nu_0(t) + \sin(\theta_0)\bar{\nu}_0(t) \]  

(34)

where \(-\pi \leq \theta_0 \leq \pi, \) and \( \bar{\nu}_0(t) \) is the Hilbert transform of \( \nu_0(t) \) for \( (i=1, 2) \). Here the spectral shaping of \( \nu_0(t) \) is based on filtering of white Gaussian noise by using an ideal filter whose transfer function is given by \( H_\nu(f) = \sqrt{S_{\nu_\rho}(f)} \). The autocorrelation functions \( R_{\mu_1\mu_1}(\tau) \) and \( R_{\mu_2\mu_2}(\tau) \) as well as the cross-correlation functions \( R_{\mu_1\nu_0}(\tau) \) and \( R_{\mu_2\nu_0}(\tau) \) can be expressed in terms of the autocorrelation function \( R_{\nu_\rho\nu_\rho}(\tau) \) of the processes \( \bar{\nu}_0(t) \) and \( \nu_0(t) \) as follows [2]

\[ R_{\mu_1\mu_1}(\tau) = R_{\mu_2\mu_2}(\tau) = R_{\nu_\rho\nu_\rho}(\tau) \]  

(35)

\[ R_{\mu_1\nu_0}(\tau) = \cos(\theta_0) R_{\nu_\rho\nu_\rho}(\tau) - \sin(\theta_0) R_{\nu_\rho\bar{\nu}_0}(\tau) \]  

(36)

\[ R_{\mu_2\nu_0}(\tau) = \cos(\theta_0) R_{\nu_\rho\nu_\rho}(\tau) + \sin(\theta_0) R_{\nu_\rho\bar{\nu}_0}(\tau) \]  

(37)
By using these equations as well as the relation
\[ R_{\mu\mu}(\tau) = R_{\mu_1\mu_1}(\tau) + R_{\mu_2\mu_2}(\tau) + j[R_{\mu_1\mu_2}(\tau) - R_{\mu_2\mu_1}(\tau)] \]  
(38)

we get
\[ R_{\mu\mu}(\tau) = 2R_{\psi_{\theta_0}}(\tau) - j2\sin\theta_0 R_{\psi_{-\theta_0}}(\tau) \]  
(39)

After Fourier transforming \( R_{\mu\mu}(\tau) \), the power spectral density can be given as
\[ S_{\mu\mu}(f) = 2S_{\psi_{\theta_0}}(f) - j2\sin\theta_0 S_{-\psi_{\theta_0}}(f) \]  
(40)

which can be expressed in terms of \( S_{\psi_{\theta_0}}(f) \) as
\[ S_{\mu\mu}(f) = 2[1 + \text{sgn}(f)\sin\theta_0]S_{\psi_{\theta_0}}(f) \]  
(41)

Measurements have shown that in many wireless communication systems the statistical behavior of log-normal fading is quite similar to a lognormal process [2]. With such a process, the slow fluctuation of the local mean value \( \lambda(t) \) of the received signal, which is determined by shadowing effects, is given by
\[ \lambda(t) = e^{\sigma_\nu^2\nu(t) + m_3} \]  
(42)

where \( \nu_3(t) \) is a real-valued Gaussian random process with mean of zero and variance of unit. The model parameters \( m_3 \) and \( \sigma_\nu \) can be used in connection with the parameters of the extended Rice process \( (\sigma_0^2, f_{max}, \kappa_0, \theta_0, \rho, \theta_\beta) \) to fit the model behavior to the statistics of real-world channels. We assume that the stochastic process \( \nu_3(t) \) is statistically independent of the process \( \nu_0(t) \), \( S_{\nu_1\nu_0}(f) \) is assumed to have the form of the Gaussian power spectral density [2], [4]:
\[ S_{\nu_1\nu_2}(f) = \frac{1}{\sqrt{2\pi\sigma_c}} e^{-\frac{f^2}{2\sigma_c^2}} \]  
(43)

the 3-dB-cut-off frequency \( f_c = \sigma_c\sqrt{2\ln 2}/\mathbb{B} \) is generally much smaller than the maximum Doppler frequency \( f_{\text{max}} \). The autocorrelation function of the process \( \nu_3(t) \) can be expressed as
\[ R_{\nu_1\nu_3}(\tau) = e^{-2(\sigma_\nu^2\tau)^2} \]  
(44)

which corresponds to the inverse Fourier transform of \( S_{\nu_1\nu_3}(f) \). The autocorrelation function \( R_{\lambda\lambda}(\tau) \) of the log-normal process can be expressed in terms of \( R_{\nu_1\nu_3}(\tau) \) as
\[ R_{\lambda\lambda}(\tau) = e^{2m_3 + \sigma_\nu^2[1 + R_{\nu_1\nu_3}(\tau)]} \]  
(45)

The power spectral density \( S_{\lambda\lambda}(f) \) can now be expressed in terms of the power spectral density \( S_{\nu_1\nu_3}(f) \) as follows
\[ S_{\lambda\lambda}(f) = e^{2m_3 + \sigma_\nu^2}[\delta(f) + \sum_{n=1}^{\infty} \frac{\sigma_\nu^2}{n!} \frac{S_{\nu_1\nu_3}(f)}{\sqrt{n}}] \]  
(46)

where \( \delta(f) \) is the Dirac-Delta function.

Stochastic multipath propagation models for indoor and out-door mobile radio channels are in general derived by employing colored Gaussian noise processes. Efficient design and realization techniques of such processes are therefore of particular importance in the area of mobile radio channel modeling [5]. Figure 9 shows the deterministic simulation model for extended Suzuki process of Type II that approximates the behavior of the stochastic reference model shown in Fig. 8. Using Eq. (33) and Eq. (34), we get
\[ \tilde{\mu}_1(t) = \sum_{n=1}^{N_1} c_{1,n} \cos(2\pi f_{1,n} t + \theta_{1,n}) \]  
(47)

\[ \tilde{\mu}_2(t) = \sum_{n=1}^{N_1} c_{1,n} \cos(2\pi f_{1,n} t + \theta_{1,n} - \theta_0) \]  
(48)

From these two equations, it can be noted that the Doppler phases \( \theta_{2,n} \) of the second deterministic process \( \tilde{\mu}_2(t) \) depend on the Doppler phases \( \theta_{1,n} \) of the first deterministic process \( \tilde{\mu}_1(t) \), because \( \theta_{2,n} = \theta_{1,n} - \theta_0 \). The complex-valued deterministic process \( \tilde{\mu}(t) = \tilde{\mu}_1(t) + \tilde{\mu}_2(t) \) can be expressed as
\[ \tilde{\mu}(t) = \sum_{n=1}^{N_1} c_{1,n} e^{j(2\pi f_{1,n} t + \theta_{1,n})} \]  
(49)

The autocorrelation functions of the processes \( \tilde{\mu}_1(t) \)

and \( \tilde{\mu}_2(t) \) can be given as:
\[ R_{\nu_1\nu_1}(\tau) = R_{\nu_2\nu_2}(\tau) = \sum_{n=1}^{N_1} \frac{c_{1,n}^2}{2} \cos(2\pi f_{1,n} \tau) \]  
(50)

And the cross-correlation functions are:
\[ R_{\nu_1\nu_2}(\tau) = R_{\nu_2\nu_1}(\tau) = \sum_{n=1}^{N_1} \frac{c_{1,n}^2}{2} \cos(2\pi f_{1,n} \tau - \theta_0) \]  
(51)
Using the Method of Equal Distances [2], [5] to calculate the Doppler coefficients $c_{1,n}$ and $f_{1,n}$, we get

$$c_{1,n} = \frac{2\sigma_0}{\sqrt{n'}} \arcsin \left( \frac{n}{N_1} \right) - \arcsin \left( \frac{n - 1}{N_1'} \right) \right)^{1/2}$$ \hspace{1cm} (52)

And the Doppler frequencies

$$f_{1,n} = \frac{f_{\text{max}}}{2N_1'} (2n - 1)$$ \hspace{1cm} (53)

Where

$$2N_1' = \frac{\pi}{\arcsin (\kappa_0)}$$ \hspace{1cm} (54)

is an auxiliary variable that depends on the frequency ratio $\kappa_0 = f_{\text{max}}/f_{\text{min}}$. The Doppler phases $\theta_{1,n}$ are assumed to be realizations of a random variable uniformly distributed within the interval $(0, 2\pi]$. The computation of the discrete Doppler coefficients $c_{1,n}$ of the deterministic Gaussian process $\nu_3(t)$, whose power spectral density is Gaussian shaped, are given by the solution to

$$c_{3,n} = \sqrt{\frac{\sigma_0^2}{\pi} \left[ \text{erf} \left( \frac{n \kappa_0 \sqrt{\ln(2)}}{N_a} \right) - \text{erf} \left( \frac{(n - 1) \kappa_0 \sqrt{\ln(2)}}{N_a} \right) \right]}$$ \hspace{1cm} (55)

Where $\text{erf}(.)$ is the error function. The Doppler frequencies $f_{3,n}$ can be computed as

$$c_{3,n} = \frac{\sqrt{2} f_{\text{max}}}{N_3} (2n - 1)$$ \hspace{1cm} (56)

The autocorrelation function of $\mu_i(t)$, for $i=1,2$, when the number of harmonic functions $N_i$ is considered as 19 (ideally $\infty$), and the maximum Doppler frequency $f_{\text{max}}$ is 85Hz. The difference between the autocorrelation function of the reference model and the simulation model decreases by increasing the number of harmonic functions ($N_i$). The power spectral density of $\nu_3(t)$ shown in Fig. 11, resembles the Gaussian power spectral density given by eq. 43. Here again the difference between the auto-correlation function of the reference model and the simulation model decreases by increasing the number of harmonic functions $N_3$. The cut-off frequency $f_c$ is selected in such a way that the mean power of the Gaussian power spectral density obtained makes up at least $99.99\%$ of its total mean power. This demand is fulfilled with $f_c = \sqrt{\ln(2)} f_{\text{max}}$ [6].

**Fig. 11:** Power spectral density $S_{\nu_3,\nu_3}(f)$ and Autocorrelation function $R_{\nu_3,\nu_3}(\tau)$ with $N_3=19$, $\sigma^2=1$, and $f_{\text{max}}=85Hz$.

**Fig. 10:** Power spectral density $S_{\mu_i,\mu_i}(f)$ and Autocorrelation function $R_{\mu_i,\mu_i}(\tau)$, for $i=1,2$, with $N_i=19$, $f_{\text{max}}=85Hz$, $\sigma^2=1$.

Figure 10 shows the power spectral density (as determined by the method of equal distances) as well as the autocorrelation function of $\mu_i(t)$, for $i=1,2$, where the number of harmonic functions $N_i$ is considered as 19 (ideally $\infty$), and the maximum Doppler frequency $f_{\text{max}}$ is 85Hz. The difference between the autocorrelation function of the reference model and the simulation model decreases by increasing the number of harmonic functions ($N_i$). The power spectral density of $\nu_3(t)$ shown in Fig. 11, resembles the Gaussian power spectral density given by eq. 43. Here again the difference between the auto-correlation function of the reference model and the simulation model decreases by increasing the number of harmonic functions $N_3$. The cut-off frequency $f_c$ is selected in such a way that the mean power of the Gaussian power spectral density obtained makes up at least $99.99\%$ of its total mean power. This demand is fulfilled with $f_c = \sqrt{\ln(2)} f_{\text{max}}$ [6].

**7. CONCLUSIONS**

In this paper, we discussed stochastic and deterministic (simulation) models for the extended Suzuki process of type I, and a comparison with extended Suzuki process of type II has been conducted. The mean-square-error method (MSEM) was used to compute the primary parameters of the simulation model (Doppler coefficients and discrete Doppler frequencies), where finite numbers of harmonics were used to simulate the short term and long term fading components of this model. As a result, it was found that the deep fades for heavy shadowing regions are much larger than that for light shadowing regions.

**References**


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