BER Performance Investigation of WPM with Unknown CSI Using Time-Domain MMSE Equalization

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ABSTRACT

Wavelet Packet Modulation (WPM) is an emerging Multi-Carrier Modulation (MCM) technique regarded as a potential alternative to widely used Orthogonal Frequency Division Multiplexing (OFDM). OFDM uses Cyclic Prefix (CP) and can rely on one-tap equalization. Since WPM has overlapping symbols in the time domain, equalization cannot depend on the use of CP. This paper considers time-domain Minimum Mean-Square Error (MMSE) equalization for WPM with unknown Channel State Information (CSI), where equalizer filter coefficients are computed using known training sequences. The influence of the transmission overhead due to training sequences on the Bit Error Rate (BER) performance is investigated considering quasistatic time-varying Rayleigh fading channels. Numerical results from computer simulations show that, with moderate overhead due to training sequences, BER performances are comparable with the case of known CSI. Furthermore, WPM is compared to OFDM with one-tap equalization, and is shown to provide better BER performances as compared to OFDM. Finally, this paper demonstrates improvement in Peak-to-Average Power Ratio (PAPR) of the transmitted signal using subcarrier combining for WPM in heterogeneous communication environments, where low-rate and high-rate devices share the same transmission resources.

Keywords: Multi-carrier modulation, Wavelet packet modulation, Minimum mean-square error, Time-domain equalization, Unknown channel state information

1. INTRODUCTION

Multi-Carrier Modulation (MCM) systems have been widely used to meet overwhelming demands of high data rates. The most extensively used MCM is Orthogonal Frequency Division Multiplexing (OFDM). However, OFDM suffers from certain drawbacks including high Peak-to-Average Power Ratio (PAPR) and wastage of transmission resources due to the use of Cyclic Prefix (CP). In light of these facts, several alternatives to OFDM have been proposed, with Wavelet Packet Modulation (WPM) as one potential candidate [1–5].

The essential features that distinguish WPM from OFDM are mentioned in [6, 7]. First, OFDM is based on Fourier transform; it uses Inverse Fast Fourier Transform (IFFT) at the transmitter and Fast Fourier Transform (FFT) at the receiver. WPM is based on a filter-tree structure; it uses Inverse Discrete Wavelet Packet Transform (IDWPT) at the transmitter and Discrete Wavelet Packet Transform (DWPT) at the receiver. Second, OFDM symbols have their length equal to the input length or the number of subcarriers, while the length of each WPM symbol is larger than the input length or the number of subcarriers, resulting in overlapping between successive WPM symbols in the time domain. Third, the output of IFFT is in general complex, so additional signal processing such as Hermitian symmetry is required to produce real signals. On the other hand, the output of IDWPT is real, making WPM favorable for applications using real signals. Fourth, IFFT is based on a fixed computational structure while IDWPT is based on a flexible iterative filter-tree structure, which allows for easy reconfiguration of the number and the bandwidths of subcarriers depending on the Channel State Information (CSI) and required data rates. Lastly, OFDM symbols are shorter and non-overlapping in the time domain, resulting in higher spectral side lobes and higher interference compared to WPM that employs overlapping symbols.

High signal PAPRs are considered as a drawback of MCM. Despite a lot of studies on PAPR reduction for OFDM, there is limited literature on PAPR reduction for WPM [6, 8–11]. The impact of different wavelet families, wavelet filters, and wavelet filter lengths is studied in [6]. The results show that the PAPR performances of different wavelet families are similar irrespective of the filter lengths. In [8], the same authors propose a PAPR reduction approach based on changing the phases of subcarriers randomly in multiple frames and selecting the combination with
the lowest PAPR. In [9], the influence of pulse shaping on PAPR is studied. The results reveal that, for low numbers of subcarriers, PAPR is reduced significantly for WPM as compared to OFDM. In [10], PAPR reduction for WPM is achieved by combining Partial Transmit Sequences (PTS) with a tree pruning approach. This method results in PAPR reduction of more than 1 dB. The PTS approach with embedded side information (ESI) to the transmit frame is proposed in [11]. This approach reduces the PAPR by 2.5 dB.

With the use of CP, equalization for OFDM can be achieved by a single-tap filter often known as one-tap equalization. However, overlapping symbols make the use of CP infeasible for WPM. Existing equalization techniques for WPM can be broadly classified as time-domain equalization [12–15] and frequency-domain equalization [7, 16]. Time-domain equalization is performed at the receiver before DWPT while frequency-domain equalization is performed after DWPT. For each type of equalization, both known CSI and unknown CSI have been considered.

For known CSI, in [12], the authors propose a time-domain equalization approach called the Minimum Square Variance (MSV) algorithm based on the variance of the demodulated output. However, as the number of subcarriers increases, the computational complexity becomes excessive. In [13], equalization based on the peak distortion criterion is presented and evaluated for various wavelet families with different filter lengths and numbers of equalizer taps. However, the approach considers only Inter-Symbol Interference (ISI) though not Inter-Carrier Interference (ICI). Time-domain MMSE equalization for WPM with known CSI is investigated in [14]. MMSE equalization is compared with frequency-domain Zero Forcing (ZF) equalization and OFDM with CP, and shown to perform the best. Frequency-domain equalization for WPM is introduced in [7]. However, it requires a complex computational structure, making it more difficult to implement as compared to time-domain equalization.

ZF and MMSE equalization with unknown CSI is investigated in [15, 16]. In [15], channel estimation is accomplished using the Least Square (LS) method and equalization is performed in the time domain. The numerical results show that BER performance with LS channel estimation is comparable to perfect estimation with known channel. However, the research is limited to static channels without considering any random fading and the proposed approach uses long training sequences of 100 symbols, resulting in high computational complexity. In [16], channel estimation using LS and MMSE criteria is proposed, to be followed by frequency-domain equalization. A pilot is used to transmit training sequences for channel estimation. The results indicate that MMSE is preferred to LS. However, the proposed method employs equalization in the frequency domain, increasing the computational complexity.

Although several literature investigates WPM with unknown CSI, the best of our knowledge, no study has been conducted to quantify the required overhead due to training sequences for time-varying channels. In this paper, for low computational complexity, time-domain MMSE equalization is used. Quasistatic time-varying Rayleigh fading channels are considered based on the models in ITU-R Rec. M.1225 [17]. This paper provides the following technical contributions.

- It quantifies the lengths of training sequences for WPM with unknown CSI to achieve acceptable BER performances compared to the case when CSI is known.
- It demonstrates using computer simulations that, with moderate overhead due to training sequences, BER performances for WPM with unknown CSI are comparable with those of known CSI.
- It compares the BER performance of WPM to OFDM with one-tap equalization, and shows that WPM can provide better BER performances as compared to OFDM.
- It proposes subcarrier combining using the flexible filter tree structure of WPM to reduce the PAPR for heterogeneous communication environments in which low-rate and high-rate devices coexist, which is not possible in OFDM.

2. SYSTEM MODEL

Fig. 1(a) shows the block diagram of a WPM system. Data bits to be transmitted are fed to a serial-to-parallel converter and, then, passed to QAM mapping to produce QAM symbols. A randomly generated yet known training sequence is inserted after every fixed number of QAM symbols. The QAM symbols along with known training sequences are input to IDWPT, which generates transmit WPM symbols. The transmitted WPM symbols go through the channel with unknown Channel Impulse Response (CIR) and Additive White Gaussian Noise (AWGN). The known training sequences are used to set equalizer filter coefficients at the receiver while the received QAM symbols are equalized in the time domain before being passed through DWPT. The output of DWPT is passed through QAM demapping and a parallel-to-serial converter to obtain the received data bits. The difference between the received and transmitted data bits will give the BER.

The transmitted WPM signal, denoted by \( s_n \), is convolved with CIR of length \( C \), denoted by \( h_n \). Assuming \( w_n \) as the complex AWGN sequence with mean zero and variance \( N_0 \), the received signal \( r_n \) can be written as

\[
r_n = s_n * h_n + w_n
\]  

where the notation \( * \) indicates the convolution operation. The received signal \( r_n \) is equalized by a linear
MMSE equalizer filter with coefficients $f_n$, resulting in the output signal

$$y_n = r_n * f_n$$

The received QAM symbols are obtained by passing $y_n$ through DWPT. Finally, received QAM symbols go through QAM demapping to produce received data bits.

### 2.1 Wavelet Packet Modulation

The basic computation unit of IDWPT and DWPT for WPM is composed of pairs of Low-Pass Filters (LPFs) and High-Pass Filters (HPFs) with symmetric decomposition of LPF and HPF outputs. This leads to uniform bandwidth distribution for subcarriers across the entire frequency range. The decomposition process is regarded as the analysis process. The inverse operation is the reconstruction process known as the synthesis process. Let $g^d_n$ and $h^d_n$ denote analysis LPF and HPF, respectively. Let $g^r_n$ and $h^r_n$ denote synthesis LPF and HPF, respectively. Let $F$ symbolize the filter length. These filter pairs are related to each other as given by [18]

$$h^d_n = (-1)^n g^d_{F-1-n}$$

$$h^r_n = h^d_{F-1-n}$$

$$g^r_n = g^d_{F-1-n}$$

Like OFDM, the inverse transform (IDWPT) is used at the transmitter while the forward transform (DWPT) is used at the receiver. Let $N$ be the number of subcarriers, which depends on the number of reconstruction stages. IDWPT is illustrated in Fig. 1(b), where the number of reconstruction stages is 2, yielding $N = 4$. More generally, $N = 2^M$, where $M$ is the number of stages. In each stage, an input signal is passed through up-sampling, which is followed by LPF or HPF, after which each pair of output signals are combined. The reverse operation takes place in DWPT, where an input signal is first passed through LPF and HPF whose outputs are down-sampled. To produce one WPM symbol, $N$ QAM data or pilot symbols are passed through IDWPT. The length of a WPM symbol generated by IDWPT is in general larger than the input length $N$, resulting in overlapping WPM symbols in the time domain. In terms of filter length $F$ and number of subcarriers $N$, the length of a WPM symbol is [18]

$$L = (N - 1)(F - 1) + 1$$

### 2.2 Linear MMSE Equalization

Equalization is performed to mitigate the impact of interference and distortion introduced by the channel. We use a linear MMSE equalizer which operates in the time domain [19]. Effectively, the same equalization filter is used for the combined signal of all subcarriers, making the approach less computational intensive compared to having separate equalizers for different subcarriers. When the CSI is unknown, a known training sequence is used, with linear equalizer coefficients estimated such that the Mean-Square Error (MSE) between the equalizer output and the known signal is minimized. The equalizer filter is given by [19]

$$f = (R^T R)^{-1} R^T s$$

In Eq. (7), $s$ is a column vector containing a delayed transmitted training sequence of symbols, $R$ is a Toeplitz matrix containing the received signals before equalization. The column vector $f$ contains $E$ equalizer coefficients which form a tapped delay line Finite Impulse Response (FIR) filter. In particular, $f$, $s$, and $R$ are given by

$$f = \begin{bmatrix} f_1 & f_2 & \cdots & f_{E-1} \end{bmatrix}$$

$$s = \begin{bmatrix} s_{E-1-\delta} \\ s_{E-\delta} \\ s_{E+1-\delta} \\ \vdots \\ s_{p-1-\delta} \end{bmatrix}$$
where $p$ is the length of the training sequence and $\delta > 0$ is the processing delay for equalization. Note that the first $E - 1$ values of the training sequence are not considered for MSE due to equalization processing delay.

On the other hand, if the CSI is known, equalizer coefficients can be derived based on [20] as given by Eq. (11). This eliminates the need of a training sequence.

\[
f = \Gamma^{-1} \xi \tag{11}
\]

In Eq. (11), $f$ is the column vector containing $E = 2K + 1$ coefficients of the equalizer, $\Gamma$ is a $(2K + 1) \times (2K + 1)$ matrix with element $g_{l,m}$ as given by Eq. (12), where $-K \leq l, m \leq K$ and $\xi$ is a column vector containing CIR as given by Eq. (13). In $\xi$, the first $K - (C - 1)$ elements and the last $K$ elements are equal to zero.

\[
g_{l,m} \triangleq \sum_{k=0}^{C-1} h_{k+m} - i h_{k}^* + N_0 \delta_{m-l} \tag{12}
\]

\[
\xi \triangleq \begin{bmatrix} h_K^* & h_{K-1}^* & \cdots & h_{-K}^* \end{bmatrix}^T = \begin{bmatrix} 0 & \cdots & 0 & h_{C-1}^* & \cdots & h_0^* \end{bmatrix}^T \tag{13}
\]

In Eq. (12), the superscript $*$ indicates the complex conjugate operation and $\delta$ represents the impulse function.

3. BER PERFORMANCE

BER performance of WPM with unknown CSI is investigated using a simulation program developed based on MATLAB. The simulation program is developed following the block diagram in Fig. 1(a). The transmit WPM frame structure is shown in Fig. 2. The overall transmit WPM symbols are separated into different blocks of equal length. For each block, a known training sequence is followed by data symbols. The simulations are conducted for both channels to ensure that the investigation results are not limited to a specific scenario and are applicable in general.

At the receiver, based on the known training sequence, equalizer coefficients are computed for each block using Eq. (7). Using the equalizer coefficients of each block, data symbols within the block are equalized in the time domain before being passed through DWPT. The BERs for each test environment are computed assuming a data rate of 10 Mbps. The other simulation parameters are listed in Table 1. Based on these parameters, the block duration in Fig. 2 is computed to be 1.28 ms, which is smaller than typical coherence times of at least 2 ms [21–23], making the quasistatic assumption reasonable.

### 3.1 BER Performance for Different Training Sequence Lengths

Simulation is executed to investigate the required lengths of known training sequences for linear MMSE equalization to achieve acceptable BER performances. Channel A of the indoor office test environment is used for specific demonstrations. The BER plots are made for different subcarriers (16, 32, and 64) with unknown CSI, and are compared with the BER plots for known CSI. The BER performance of the MMSE equalizer with unknown CSI relies on the number of signal values employed for the training. A sufficiently long training sequence yields the BER performance comparable to the case with known CSI, as demonstrated by the simulation results in the
The BER plots for 16 subcarriers are shown in Fig. 3. The BER plots show that, with a training length of 1 WPM symbol, BERs are significantly higher than those for known CSI. As the training sequence length increases to 2, 3, and 4 WPM symbols, BERs gradually decrease. Eventually, with the training sequence length of 5 WPM symbols, BER performances for unknown CSI are comparable to those for known CSI. The BER plots for 32 and 64 subcarriers are shown in Figs. 4–5. The numerical results show that, for 32 subcarriers, a training sequence length of 2 WPM symbols are required while, for 64 subcarriers, a training sequence length of 1 WPM symbol is sufficient. When the number of subcarriers increases, each WPM symbol becomes longer, i.e., contains more signal values. Therefore, given the same number of WPM symbols in the training sequence, the training sequence length in terms of the number of signal values is higher for a higher number of subcarriers, and hence a better BER performance. For example, using one WPM training symbol, there are 64 signal values in the training sequence with 64 subcarriers, but only 32 signal values in the training sequence with 32 subcarriers.

Training sequence lengths to achieve acceptable BER performances for channel A and channel B of all the test environments for N = 16, N = 32, and N = 64 are shown in Table 2. The maximum training sequence length for N = 16 is 5 WPM symbols. Similarly, the maximum training sequence length for N = 32 is 2 WPM symbols, while for N = 64, the training sequence length of 1 WPM symbol is sufficient in all cases. With a block size of 100 WPM symbols, the overhead due to training sequences are at most 5% in all scenarios, and at most 1% when N = 64.

Table 2: Training sequence lengths (in number of WPM symbols) required to achieve acceptable BER performances for channel A and channel B of each test environment for N = 16, N = 32, and N = 64

<table>
<thead>
<tr>
<th>Test environment</th>
<th>Channel</th>
<th>Training sequence length</th>
<th>N=16</th>
<th>N=32</th>
<th>N=64</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indoor office</td>
<td>A</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Outdoor-to-indoor &amp; pedestrian</td>
<td>A</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Vehicular</td>
<td>A</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
3.2 BER Comparison between WPM and OFDM

Simulation programs were developed to compare the BER performances of WPM and OFDM with 64 subcarriers. The BER plots are obtained for WPM with known CSI, WPM with unknown CSI, OFDM with known CSI and OFDM with unknown CSI. Channel estimation for OFDM with unknown CSI is accomplished using LS channel estimation as given in [24], followed by one-tap equalization. The training sequence length of 1 WPM symbol is used for WPM with unknown CSI. BER plots for channel A and channel B of different test environments are shown in Figs. 6-8. The numerical results for all the test environments verify that the BERs of OFDM for unknown CSI are higher compared to known CSI. From the BER plots for both channels of all test environments, WPM performs better than OFDM for both known and unknown CSI. The reductions of $E_b/N_0$ for WPM compared to OFDM to achieve a BER of $10^{-4}$ are listed in Table 3. A maximum $E_b/N_0$ improvement of 2.5 dB is obtained for channel A of vehicular test environment while the minimum $E_b/N_0$ improvement of 1 dB is observed for channel B of indoor office test environment. Overall, the results indicate that WPM with moderate lengths of training sequences can achieve better BER performance than OFDM.

The overhead due to training sequences for WPM is modest compared to the overhead for OFDM. The numerical results show that, for $N = 64$, a training sequence length of one WPM symbol is sufficient. For OFDM, the minimum training sequence length of 1 OFDM symbol is required for channel estimation. In addition, there will be overhead due to CP in each OFDM symbol. Overall, the overhead for OFDM, in general, will be larger.
4. PAPR FOR WPM WITH SUBCARRIER COMBINING

Computation of IDWPT and DWPT for WPM are based on a filter-tree structure. Flexibility of the filter-tree structure can be used to improve the PAPR in heterogeneous communication environments, where devices operating with high data rates and low data rates share the same transmission resources. In such a scenario, the devices operating with high data rates can be assigned larger numbers of subcarriers. For simplicity, an example with 16 subcarriers is illustrated, considering three devices with high data rates and four devices with low data rates. Fig. 9(a) shows the time-frequency structure without subcarrier combining and with equal bandwidth allocation to all subcarriers. The corresponding filter-tree structure of WPM is shown in Fig. 9(b). This resource utilization is similar to the case of OFDM with IFFT and FFT as the forward and inverse transforms.

In case of WPM, subcarrier combining can be re-

Table 3: Reductions of $E_b/N_0$ for WPM compared to OFDM to achieve BER of $10^{-4}$

<table>
<thead>
<tr>
<th>Test environment</th>
<th>Channel</th>
<th>$E_b/N_0$ reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indoor office</td>
<td>A</td>
<td>1.5 dB</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>1 dB</td>
</tr>
<tr>
<td>Outdoor-to-indoor and</td>
<td>A</td>
<td>2 dB</td>
</tr>
<tr>
<td>pedestrian</td>
<td>B</td>
<td>1.5 dB</td>
</tr>
<tr>
<td>Vehicular</td>
<td>A</td>
<td>2.5 dB</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>1.5 dB</td>
</tr>
</tbody>
</table>
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Fig. 10: (a) Time-frequency structure and (b) tree structure, of WPM for 16 subcarriers with subcarrier combining

alized, resulting in a modified time-frequency structure. However, this subcarrier combining is not feasible in case of OFDM. Subcarrier combining for WPM yields a higher bandwidth with shorter symbol durations. Subcarrier combining results in reducing the number of subcarriers and ultimately reducing the PAPR. Following the example in Fig. 9, consider that four subcarriers are allocated to each of the three high-rate devices, and one subcarrier to each of the four low-rate devices. Each group of four subcarriers allocated to a high-rate device are combined into a single combined subcarrier. The corresponding time-frequency structure and filter-tree structure are shown in Fig. 10.

Simulation is performed to compare the cumulative distribution functions (CDFs) of PAPR for WPM without subcarrier combining, WPM with subcarrier combining and OFDM. CDF plots are obtained using 64 subcarriers and shown in Fig. 11. Although the PAPRs for WPM without subcarrier combining and OFDM are almost the same, WPM with subcarrier combining has significantly lower PAPRs. The results reflect that the subcarrier combining feature in WPM is advantageous in reducing the PAPR for heterogeneous communication environments.

Fig. 11: CDF plot for PAPR of WPM without subcarrier combining, WPM with subcarrier combining, and OFDM

5. CONCLUSION

BER performances of WPM with unknown CSI were investigated to study the impact of the training sequence overhead on the BER. For performance evaluation, two channel models of three different test environments according to ITU-R M. 1225 are considered. Simulation results show that, with moderate overhead due to training sequences, BER performances are nearly the same as those for known CSI. In particular, the required overhead of training sequences for 16, 32, and 64 subcarriers are at most 5%, 2%, and 1% respectively. In addition, with unknown CSI, WPM performs better than OFDM for all test scenarios. Channel A of vehicular test environment corresponds to the largest $E_b/N_0$ reduction of 2.5 dB, while channel B of indoor office test environment corresponds to the smallest $E_b/N_0$ reduction of 1 dB. Besides the BER improvement, the paper demonstrates that, for heterogeneous communication environments, the subcarrier combining feature in WPM can reduce the PAPR of the transmit signal as compared to WPM without subcarrier combining and OFDM. For future research, channel estimation and bit loading for WPM with unknown CSI can be investigated.

6. ACKNOWLEDGMENTS

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References


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